Kashmir Journal of Science
https:\\kjs.org.pk

Kashmir Journal of Science (2022), 1(1): 68-76

Research Paper

Survival analysis based on doubly type-II censored samples

Shaila Qadeem[1], Rijah Khan[2*] and Sayyad Khurshid [3]

[1]*Government Girls Elementary School Bandi Kokyal, Muzaffarabad Azad Jammu & Kashmir*
[2]*School of Natural Sciences (SNS), National University of Sciences and Technology (NUST), Islamabad, Pakistan.*
[3]*Captain Hussain Khan Shaheed, Government Boys Post Graduate College Rawalakot, Azad Kashmir, Pakistan*
**Corresponding author's email:* rijahkhan2013@gmail.com

A R T I C L E I N F O

Article history:

Received: 21 September 2022
Accepted: 20 October 2022
Available online: 02-11-2022

Keywords:

Bayesian estimator
Frechet distribution
Maximum likelihood estimator
Type-II censored sample

Abstract

This study emphasis the Bayesian analysis for survival data using Frechet distribution based on doubly type-II censored samples, when some of the sample values might have been contaminated at either or both extremes, because in a clinical disease the maturation time of disease cannot predict exactly. The Bayesian estimators based on reference prior are proposed and performances of these estimators are compared with maximum likelihood estimators via simulation study based on standard deviation by taking different sample sizes and different parametric values. A real data set about relief times (in minutes) of 20 patients receiving an analgesic is analyzed for illustrative purposes.

Introduction

From time to time data cannot be recorded or collected accurately due to unanticipated situations. Therefore, to overcome such situation doubly type-II censoring scheme can be adopted and conventional procedures can be used for estimating the parameters of lifetime distributions under doubly type-II censoring scheme. Doubly type-II censored sampling is a well-known technique of obtaining data in many lifetime studies. Doubly censored samples are extensively used in various dimension of statistical practice, particularly when some of the sample values might have been contaminated at either or both extremes. Doubly-censored data usually come up in survival studies in which both the originating and failure times are censored.

In this paper, we propose Bayesian and maximum likelihood estimators for two parameters Frechet distribution (1927) based on doubly type-II censored scheme when the lifetime observations are imprecise quantities**.** Frechet distribution (FD) has a significant role in reliability engineering, network engineering, space engineering and nuclear engineering. It can also be used to check the variety of failure characteristics such as infant mortality. The probability density function (PDF) of two parameters FD is

Survival analysis based on doubly type-II censored samples

$$f(x;\alpha,\beta)=\frac{\alpha}{\beta}\left(\frac{\beta}{x}\right)^{\alpha+1} exp\left[-\left(\frac{\beta}{x}\right)^{\alpha}\right], x>0, \alpha,\beta>0\ , \tag{1}$$

The corresponding cumulative distribution function (CDF)

$$F(x)=exp\left[-\left(\frac{\beta}{x}\right)^{\alpha}\right], \tag{2}$$

where α indicates the shape of distribution and β is scale parameter. Lots of work has been done on the parameter estimation of FD by using classical and Bayesian methods. Estimators for FD have also been developed under type-II censoring scheme. However, no work has been done on FD under doubly type-II censoring samples. In the present study, we have proposed the Maximum likelihood (ML) and Bayesian estimators for FD using doubly type-II censored data.

Some of the earlier work based on doubly type-II censored samples using different distribution and also parameter estimation of FD was conducted. For example, Fernandez (2000) discussed Bayesian inference from type-II doubly censored Rayleigh data. Lin and Balakrishnan (2003) obtained exact prediction intervals for failure times from one parameter and two parameter exponential distributions based on doubly type-II censored samples. Wu (2008) studied interval estimation for Pareto distribution having doubly type-II censored sample. Kim and Song (2010) considered the problem of estimating parameters and reliability function of the generalized exponential distribution, based on doubly type-II censored sample using Bayesian viewpoints.
Extensive work on parameter estimation of FD has been found in Abbas and Tang (2012, 2013, 2014, 2015). Moreover, Pak et al. (2013) estimated the Rayleigh scale parameter under doubly type-II censoring from imprecise data.

The main objective of this paper is to estimate the parameters of FD based on doubly type-II censored data using the Bayesian and ML method. The originality of this study comes from the fact that, for the FD, there has been no previous work considering data with doubly type-II censoring mechanisms. Including this introduction, the rest of the paper is organized as follows. ML and Bayesian estimation procedure are presented in Section 2 and Section 3 respectively. In Section 4, simulation study is presented. For illustration a real data is analyzed in Section 5 and Section 6 comprised the concluding remarks.

Maximum likelihood estimation

Suppose $x_{(r)}, x_{(r+1),\dots,} x_{(n-s)}$ be a random sample of size n from FD, where these are ordered observations that can be only examined. In failure censored experiment a known number of values are observed and some values are missing at the both extreme. Let (r-1) smallest and (n-s) largest observations are assumed to be censored, then the likelihood function can be expressed as

$$l(x;\alpha,\beta)=\frac{n!}{(r-1)!(n-s)!}\prod_{i=r}^{s} f(x_{(i)};\alpha,\beta)\left[F\left(x_{(r)};\alpha,\beta\right)\right]^{r-1}\left[1-F\left(x_{(s)};\alpha,\beta\right)\right]^{n-s}, \tag{3}$$

$$l(x;\alpha,\beta)=\frac{n!}{(r-1)!(n-s)!}\prod_{i=r}^{s}\left[\frac{\alpha}{\beta}\left(\frac{\beta}{x_{(i)}}\right)^{\alpha+1} exp\left[-\left(\frac{\beta}{x_{(i)}}\right)^{\alpha}\right]\right]\left[exp\left[-\left(\frac{\beta}{x_{(r)}}\right)^{\alpha}\right]\right]^{r-1}\left[1-exp\left[-\left(\frac{\beta}{x_{(s)}}\right)^{\alpha}\right]\right]^{n-s} \tag{4}$$

It is more convenient to work with log-likelihood function. The log-likelihood function is

Survival analysis based on doubly type-II censored samples

$$L = \ln(w) + (s+r+1) + \alpha(s-r+1)\ln(\beta) - (\alpha+1)\sum_{i=r}^{s}\ln x_{(i)} - \sum_{i=r}^{s}\left(\frac{\beta}{x_{(i)}}\right)^{\alpha} - (r-1)\left(\frac{\beta}{x_{(r)}}\right)^{\alpha} + (n-s)\ln\left[1 - exp\left[-\left(\frac{\beta}{x_{(s)}}\right)^{\alpha}\right]\right]. \quad (5)$$

Where $L = l(x;\alpha,\beta)$ and $w\frac{n!}{(r-1)!(n-s)!}$. To get the ML estimates of α and β based on doubly type-II censored sample. The score equations are

$$\frac{\partial L}{\partial \alpha} = \frac{s-r+1}{\alpha} + (s-r+1)\ln(\beta) - \sum_{i=r}^{s}\ln x_{(i)} - \sum_{i=r}^{s}\left(\frac{\beta}{x_{(i)}}\right)^{\alpha}\ln\left(\frac{\beta}{x_{(i)}}\right) - (r-1)\left(\frac{\beta}{x_{(r)}}\right)\ln\left(\frac{\beta}{x_{(r)}}\right) + \frac{(n-s)exp\left[-\left(\frac{\beta}{x_{(s)}}\right)^{\alpha}\right]\left(\frac{\beta}{x_{(s)}}\right)^{\alpha}\ln\left(\frac{\beta}{x_{(s)}}\right)}{\left[1-exp\left[-\left(\frac{\beta}{x_{(s)}}\right)^{\alpha}\right]\right]} = 0, \quad (6)$$

$$\frac{\partial L}{\partial \beta} = \frac{(s-r+1)\alpha}{\beta} - \frac{\alpha}{x_{(i)}}\sum_{i=r}^{s}\left(\frac{\beta}{x_{(i)}}\right)^{\alpha-1} - \frac{(r-1)\alpha}{x_{(r)}}\left(\frac{\beta}{x_{(r)}}\right)^{\alpha-1} + \frac{\alpha(n-s)exp\left[-\left(\frac{\beta}{x_{(s)}}\right)^{\alpha}\right]\left(\frac{\beta}{x_{(s)}}\right)^{\alpha-1}}{x_{(s)}\left[1-exp\left[-\left(\frac{\beta}{x_{(s)}}\right)^{\alpha}\right]\right]} = 0, \quad (7)$$

However, the above equations cannot be written in closed form. Therefore, any iterative procedure can be used to get the approximate ML estimates of α and β.

Bayesian estimation

The Bayesian approach uses both sample and prior information into analysis, which will improve the quality of the inferences. In this section, Bayesian estimators are obtained in the case of doubly type-II censored samples using reference prior. As Abbas and Tang (2015) developed reference prior for FD, which is

$$\pi_R(\alpha,\beta) = \frac{1}{\alpha\beta}, \quad (8)$$

Combining equation (4) and equation (8), the joint posterior density of α and β can be written as

$$\pi(\alpha,\beta|\underline{x}) \propto \prod_{i=r}^{s}\left[\frac{\alpha}{\beta}\left(\frac{\beta}{x_{(i)}}\right)^{\alpha+1} exp\left[-\left(\frac{\beta}{x_{(i)}}\right)^{\alpha}\right]\right]\left[exp\left[-\left(\frac{\beta}{x_{(r)}}\right)^{\alpha}\right]\right]^{r-1}\left[1 - exp\left[-\left(\frac{\beta}{x_{(s)}}\right)^{\alpha}\right]\right]^{n-s}\left(\frac{1}{\alpha\beta}\right) \quad (9)$$

Generally, it is not possible for (9) to have a closed form. However, Laplace approximation can be utilized to get the Bayesian estimates of α and β.

Simulation study

Simulation study is conducted to assess the performance of ML and Bayesian estimators. The following procedure is adopted:

1. Generate a random sample of size 10, 20, 40, 60, 80, and 100 from FD using inverse transformation technique i.e., $x = \beta[-\log u_i]^{-\frac{1}{\alpha}}$, where u_i is uniformly distributed i. e., $u_i \sim (0,1)$ with (α; β) = 1, 1.5, 2, 3 and different values of r and s.

2. Using the values obtained in step 1, calculate the ML and Bayesian estimates of α and β and their standard deviations (SD) using pre-specified different percentages of 's' i.e., 50%, 750% and 90% respectively.
3. Repeat the step 1 and step 2 N times. The results are presented in Table 1-8. From the results of simulation study conclusion are summarized below.

- It is observed that the performances of all estimators become better when the sample size and the values of 'r' and 's' increase, and for large sample sizes, the Bayesian and ML estimates become closer in terms of SD. Shortly, SD decreases when 'n' is increasing.
- For estimating a, the Bayesian estimators have the smaller SD than the ML estimator. It is recommended that one should use the Bayesian estimator because it provides the smallest SD for all sample sizes in case of doubly type-II censoring. Since the Bayesian estimator outperforms than ML estimators in terms of SD.
- For β, ML estimators have lower SD than the Bayesian estimators. However, as sample size increases the parameters approaching to the true value which shows that estimators are consistent.
- In general, Laplace approximation works the best even in small sample sizes. Therefore, we recommend the Bayesian estimators based on reference prior and ML estimators are the right choice for estimating the parameters of FD in case of doubly type-II censored samples.

Table 1: Average ML and Bayesian estimates with their corresponding SD when α= 1.

n	s	r	ML	SD	Bayesian	SD
10	5	2	1.7478	0.8758	1.7380	1.1680
	7		1.7323	0.5378	1.2532	0.5343
	9		1.2456	0.3899	1.1567	0.3907
20	10	2	1.2058	0.3392	1.1628	0.3481
	15		1.1108	0.2420	1.0770	0.2446
	18		1.0825	0.2158	1.0539	0.2175
40	20	3	1.0910	0.2096	1.0699	0.2118
	30		1.0557	0.1602	1.0400	0.1611
	36		1.0411	0.1411	1.0282	0.1417
60	30	3	1.0594	0.1595	1.0473	0.1607
	45		1.0404	0.1253	1.0310	0.1258
	54		1.0180	0.1093	1.0104	0.1096
80	40	5	1.0441	0.1404	1.0337	0.1411
	65		1.0237	0.1047	1.0166	0.1050
	72		1.0191	0.0962	1.0129	0.0964
100	50	5	1.0287	0.1210	1.0211	0.1215
	75		1.0225	0.0959	1.0168	0.0961
	90		1.0172	0.0846	1.0125	0.0848

Table 2: Average ML and Bayesian estimates with their corresponding SD when α= 1.5.

n	s	r	ML	SD	Bayesian	SD
10	5	2	2.5946	1.3033	1.8537	1.1206
	7		2.0767	0.7655	1.6913	0.6970
	9		1.8458	0.5864	1.5884	0.5379
20	10	2	1.8181	0.5071	1.6487	0.4969
	15		1.6777	0.3666	1.5719	0.3608
	18		1.6314	0.3252	1.5457	0.3203
40	20	3	1.6296	0.3131	1.5580	0.3109
	30		1.5792	0.2397	1.5326	0.2383
	36		1.5565	0.2114	1.5190	0.2101

60	30	3	1.5950	0.2402	1.5532	0.2397
	45		1.5564	0.1875	1.5280	0.1870
	54		1.5312	0.1645	1.5087	0.1640
80	40	5	1.5623	0.2101	1.5288	0.2095
	65		1.5353	0.1571	1.5145	0.1567
	72		1.5271	0.1442	1.5094	0.1438
100	50	5	1.5418	0.1815	1.5177	0.1812
	75		1.5334	0.1438	1.5164	0.1436
	90		1.5250	0.1269	1.5115	0.1266

Table 3: Average ML and Bayesian estimates with their corresponding SD when α= 2.

n	s	r	ML	SD	Bayesian	SD
10	5	2	3.4595	1.7377	1.8512	1.0742
	7		2.7690	1.0207	1.9611	0.8133
	9		2.4611	0.7819	1.9348	0.6565
20	10	2	2.4241	0.6762	2.0371	0.6251
	15		2.2369	0.4888	2.0087	0.4653
	18		2.1753	0.4336	1.9920	0.4151
40	20	3	2.1728	0.4175	2.0119	0.4054
	30		2.1056	0.3196	2.0047	0.3133
	36		2.0753	0.2818	1.9952	0.2767
60	30	3	2.1267	0.3203	2.0317	0.3156
	45		2.0752	0.2500	2.0133	0.2473
	54		2.0416	0.2194	1.9931	0.2171
80	40	5	2.0831	0.2801	2.0083	0.2765
	65		2.0470	0.2094	2.0025	0.2076
	72		2.0361	0.1923	1.9984	0.1907
100	50	5	2.0557	0.2402	2.0001	0.2398
	75		2.0445	0.1918	2.0077	0.1905
	90		2.0333	0.1692	2.0043	0.1681

Table 4: Average ML and Bayesian estimates with their corresponding SD when α= 3.

n	s	r	ML	SD	Bayesian	SD
10	5	2	5.5858	3.1965	1.7915	1.0207
	7		4.1496	1.5348	2.2119	0.9345
	9		3.7388	1.1667	2.4126	0.6565
20	10	2	3.5816	1.0015	2.5366	0.8057
	15		3.3585	0.7353	2.7294	0.6459
	18		3.2514	0.6481	2.7487	0.5805
40	20	3	3.2709	0.6286	2.8042	0.5776
	30		3.1533	0.4783	2.8671	0.4530
	36		3.1172	0.4233	2.8903	0.4033
60	30	3	3.1829	0.4799	2.9026	0.5482
	45		3.0999	0.3729	2.9234	0.3614
	54		3.0724	0.3299	2.9324	0.3207
80	40	5	3.1487	0.4238	2.9266	0.4080
	65		3.0832	0.2878	2.9545	0.3080
	72		3.0475	0.1356	2.9398	0.2814
100	50	5	3.0881	0.3634	2.9234	0.3538
	75		3.0658	0.2879	2.9592	0.2820
	90		3.0552	0.2544	2.9712	0.2500

Table 5: Average ML and Bayesian estimates with their corresponding SD when β= 1.

n	s	r	ML	SD	Bayesian	SD
10	5	2	1.0499	0.4085	0.9823	0.4432
	7		1.1027	0.3985	1.0080	0.4207

	9		1.1353	0.3950	1.0286	0.4197
20	10	2	1.0361	0.2819	1.0012	0.2880
	15		1.0568	0.2621	1.0132	0.2631
	18		1.0518	0.2603	1.0057	0.2600
40	20	3	1.0157	0.1880	0.9980	0.1894
	30		1.0205	0.1756	0.9992	0.1756
	36		1.0250	0.1745	1.0023	0.1740
60	30	3	1.0015	0.1508	0.9897	0.1511
	45		1.0184	0.1421	1.0045	0.1417
	54		1.0220	0.1419	1.0071	0.1417
80	40	5	1.0092	0.1319	1.0002	0.1323
	65		1.0129	0.1220	1.0020	0.1219
	72		1.0106	0.1204	0.9993	0.1202
100	50	5	1.0048	0.1179	0.9976	0.1181
	75		1.0100	0.1091	1.0016	0.1091
	90		1.0114	0.1077	1.0025	0.1075

Table 6: Average ML and Bayesian estimates with their corresponding SD when β= 1.5

n	s	r	ML	SD	Bayesian	SD
10	5	2	1.5678	0.3745	1.5278	0.4924
	7		1.5673	0.3604	1.4863	0.4122
	9		1.5809	0.3538	1.4836	0.4025
20	10	2	1.5155	0.2607	1.4992	0.2840
	15		1.5556	0.2510	1.5208	0.2638
	18		1.5456	0.2481	1.5066	0.2581
40	20	3	1.5107	0.1844	1.5021	0.1918
	30		1.5138	0.1720	1.4969	0.1744
	36		1.5201	0.1714	1.5007	0.1176
60	30	3	1.5012	0.1483	1.4956	0.1518
	45		1.5162	0.1399	1.5055	0.1418
	54		1.5183	0.1393	1.5059	0.1408
80	40	5	1.5051	0.1298	1.5007	0.1323
	65		1.5111	0.1205	1.5023	0.1217
	72		1.5064	0.1194	1.4968	0.1203
100	50	5	1.5026	0.1165	1.4991	0.1182
	75		1.5087	0.1081	1.5022	0.1090
	90		1.5092	0.1064	1.5017	0.1071

Table 7: Average ML and Bayesian estimates with their corresponding SD when β= 2

n	S	r	ML	SD	Bayesian	SD
10	5	2	2.0555	0.3555	2.0520	0.5506
	7		2.0562	0.3470	1.9856	0.4419
	9		2.0691	0.3403	1.9771	0.4213
20	10	2	2.0093	0.2539	2.0159	0.2973
	15		2.0503	0.2428	2.0261	0.2624
	18		2.0390	0.2453	2.0088	0.2693
40	20	3	2.0078	0.1822	2.0110	0.1963
	30		2.1056	0.1705	1.9999	0.1783
	36		2.0177	0.1698	2.0024	0.1759
60	30	3	1.9994	0.1473	2.0016	0.1542
	45		2.0146	0.1389	2.0079	0.1428
	54		2.0166	0.1383	2.0072	0.1414
80	40	5	2.0037	0.1290	2.00053	0.1338
	65		2.0099	0.1199	2.0036	0.1209
	72		2.0052	0.1189	1.9977	0.1223
100	50	5	2.0014	0.1160	2.0027	0.1192

	75		2.0077	0.1077	2.0036	0.1095
	90		2.0083	0.1060	2.0026	0.1074

Table 8: Average ML and Bayesian estimates with their corresponding SD when β= 3

n	S	r	ML	SD	Bayesian	SD
10	5	2	3.0198	0.3346	3.1047	0.7258
	7		3.0580	0.3262	3.0031	0.4781
	9		3.0627	0.3289	2.9750	0.4554
20	10	2	3.0078	0.2525	3.0718	0.3444
	15		3.0254	0.2392	3.0249	0.2866
	18		3.0360	0.2377	3.0225	0.2794
40	20	3	3.0029	0.1809	3.0353	0.2102
	30		3.0126	0.1700	3.0147	0.1861
	36		3.0128	0.1679	3.0073	0.1806
60	30	3	2.9980	0.1470	3.0198	0.1614
	45		3.0106	0.1385	3.0136	0.1466
	54		3.0078	0.1367	3.0058	0.1431
80	40	5	3.0005	0.1277	3.0167	0.1376
	65		3.0096	0.1189	3.0097	0.1239
	72		3.0003	0.1187	2.9979	0.1229
100	50	5	2.9978	0.1153	3.0108	0.1220
	75		3.0015	0.1072	3.0032	0.1110
	90		3.0046	0.1054	3.0031	0.1084

Real Data Analysis

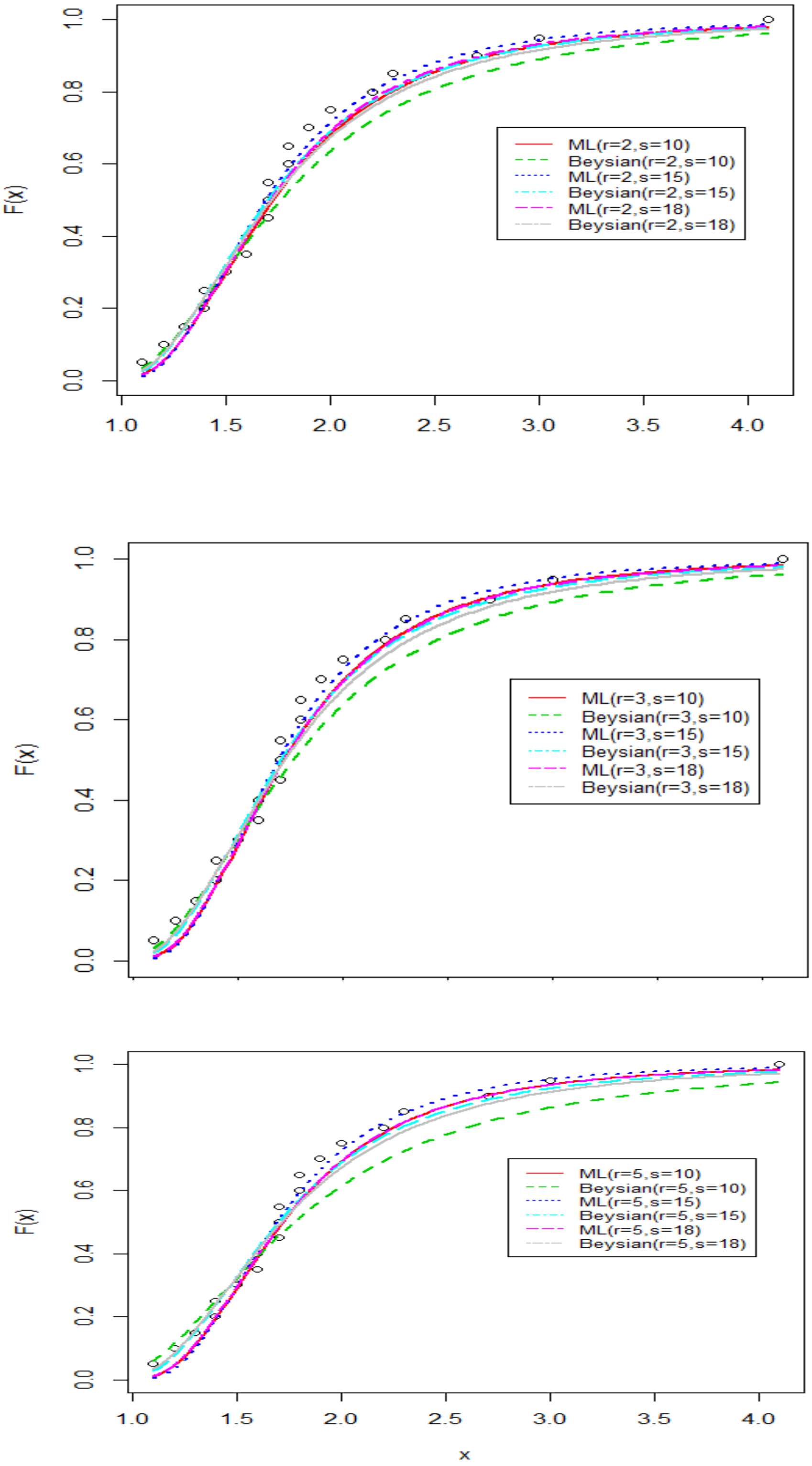
In this section, we analyze the real data relating to relief times (in minutes) of 20 patients receiving an analgesic and reported by Gross and Clark (1975) presented in Table 9. The Bayesian and ML estimates along with their SD are presented in Table 10 considering different values of "r" and "s". In this case, all the estimates are close to ML estimates. Further, Figure 1 shows the results of estimation methods considering various values of "r" and "s".

Table 9: Lifetime's data relating to relief times (in minutes) of 20 patients receiving an analgesic

1.1	1.4	1.3	1.7	1.9	1.8	1.6	2.2	1.7	2.7
4.1	1.8	1.5	1.2	1.4	3	1.7	2.3	1.6	2

Table 10: ML and Bayesian estimates for Lifetime's data.

ML						Bayesian			
R	**S**	**α**	**SD**	**β**	**SD**	**α**	**SD**	**β**	**SD**
2	10	3.9851	1.0438	1.5736	0.1025	3.3499	0.9776	1.5792	0.1224
	15	4.3275	0.8929	1.5540	0.0867	3.8628	0.8551	1.5468	0.0972
	18	4.0654	0.7812	1.5660	0.0916	3.6961	0.7488	1.5548	0.1010
3	10	4.2937	1.2436	1.5776	0.0954	3.4048	1.1121	1.5824	0.1208
	15	4.6126	1.0257	1.5646	0.0825	3.9945	0.9573	1.5548	0.0957
	18	4.2395	0.8697	1.5764	0.0894	3.7724	0.8167	1.5622	0.1010
5	10	4.2068	1.4877	1.5762	0.0981	2.9400	1.2063	1.5677	0.1413
	15	4.6388	1.1743	1.5654	0.0848	3.8069	1.0501	1.5434	0.1049
	18	4.1846	0.9541	1.5728	0.0940	3.6000	0.8703	1.5467	0.1110



(a)

(b)

(c)

Figure 1: Empirical and fitted CDFs using different methods of estimation.

Conclusion

In this paper, we introduced the Bayesian and ML estimators of FD based on doubly type-II censored samples. Bayesian estimators are developed based on reference prior and compared with ML estimators. Bayesian and ML estimators cannot be obtained in explicit form. However, Laplace approximation is utilized to obtain the approximate estimates. Simulations and real data analysis illustrate the performance of Bayesian and ML estimators in terms of standard deviations.

Simulations and analysis of real data have been used to demonstrate how the proposed estimator's work. Further, it is worth pointing that Laplace's approximation performs well even in small sample sizes and we do recommend that for studying FD in case of doubly type-II censored samples, both Bayesian and ML estimators can be the precise alternatives.

References

Abbas, K. and Tang, Y. (2015). Analysis of Frechet distribution using reference priors. Communications in Statistics—Theory and Methods, 44 (2015), 2945-2956.

Abbas, K. and Tang, Y. (2014). Objective Bayesian analysis of the Frechet stress-strength model. Statistics and Probability Letters, 84, 169-174.

Abbas, K. and Tang, Y. (2013). Estimation of parameters for Frechet distribution based on type-II censored samples. Caspian Journal of Applied Sciences Research. 2(7), 36-43.

Abbas, K. and Tang, Y. (2012). Comparison of estimation methods for Frechet distribution with known shape. Caspian Journal of Applied Sciences Research. 1 (10), 58-64.

Frechet, M., (1927), "Sur la loi de probabilite de l'ecart maximum", Ann. Soc. Math. Polon., 6, 93-116.

Fernandez, A. J. (2000). Bayesian inference from type-II doubly censored Rayleigh data. Statistics and Probability Letters, 48 (4), 393-399.

Gross A, J. and Clark, V. A. (1975). Survival Distributions: Reliability Applications in the Biometrical Sciences, John Wiley, New York, USA.

Kim, C. and Song, S. (2010). Bayesian estimation of the parameters of the generalized exponential distribution from doubly censored samples. Statistical Papers, 51 (3), 583-597.

Lin, C. T. and Balakrishnan, N. (2003). Exact prediction intervals for exponential distributions based on doubly type-II censored samples. Journal of Applied Statistics, 30, 783-801.

Pak, A. Parham, G. A. and Saraj, M. (2013). On estimation of Rayleigh scale parameter under doubly Type-II censoring from imprecise data. Journal of Data Science, 11, 305-322.

Wu, S. F. (2008). Interval estimation for a Pareto distribution based on a doubly type-II censored sample, Computational Statistics and Data Analysis, 52, 3779-3788.